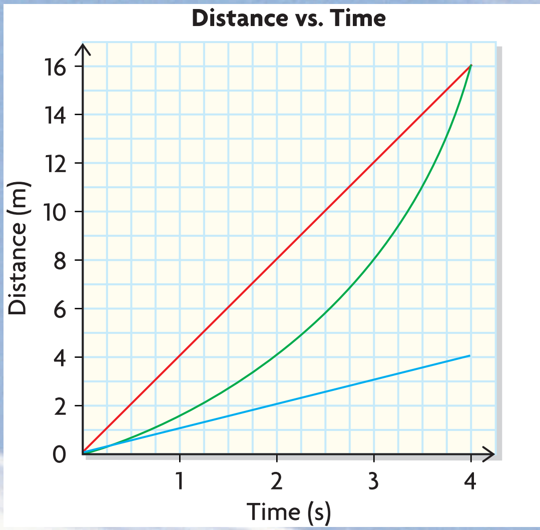


Chapter
3
Linear Relations

GOALS

You will be able to

- Recognize direct and partial variation from a graph, a table of values, and an algebraic expression
- Describe properties of linear relations
- Recognize whether a relation is linear or nonlinear from a table of values, a graph, an algebraic expression, or a written description
- Create different representations of linear and nonlinear relations

? Which of these relationships between distance and time best describes the distance the sled covers in 4 s?

3 Getting Started

Study Aid

- For more help and practice, see Appendix A-11.

WORDS YOU NEED to Know

1. Match each term with the most appropriate item.

a) scatter plot **c)** pattern rule **e)** variable
b) table of values **d)** geometric pattern **f)** data point

i)

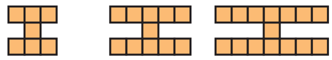
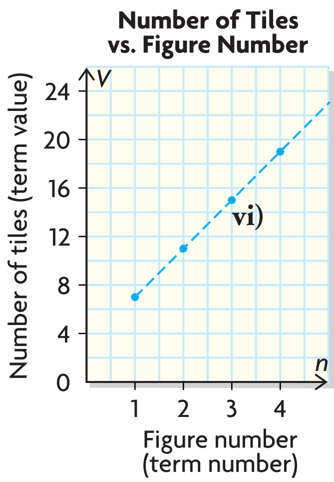


figure 1 figure 2 figure 3

ii)

Figure Number (term number)	Number of Tiles (term value)
1	7
2	11
3	15
4	19
iii) *n*	**iv)** $4n + 3$

v)



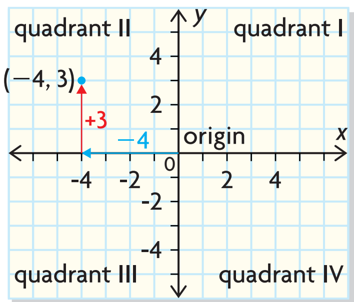
Study Aid

- For more help and practice, see Appendix A-12.

SKILLS AND CONCEPTS You Need

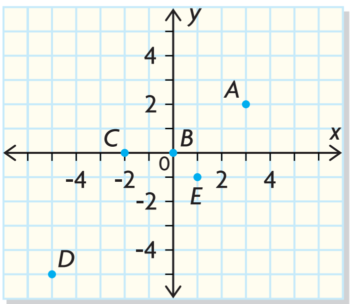
Using the Cartesian Coordinate System

The Cartesian coordinate system describes the location of a point in relation to a horizontal number line (the x-axis) and a vertical number line (the y-axis). The intersection of the axes creates four quadrants.



EXAMPLE

State the coordinates of each point.



Solution

$A(3, 2)$
$B(0, 0)$
$C(-2, 0)$
$D(-5, -5)$
$E(1, -1)$

2. Plot the following points.

a) $A(4, 6)$ **b)** $B(0, 2)$ **c)** $C(-2, -5)$

Construct a Table of Values and a Scatter Plot to Represent a Relation

Study Aid

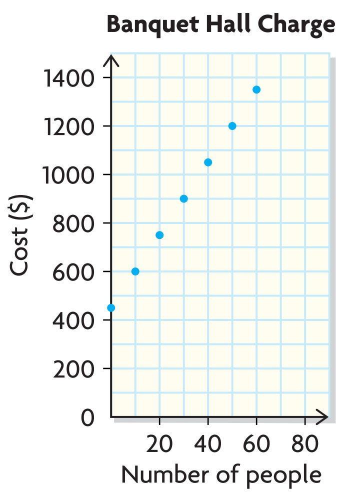
- For more help and practice, see Appendix A-13.

EXAMPLE

The cost of a banquet at Juan's Banquet Hall is \$450 for the room rental, plus \$15 for each person served. Create a table of values and a scatter plot to represent the relation between the number of people and the cost.

Solution

People	Cost (\$)
0	450 + 15(0) = 450
10	450 + 15(10) = 600
20	450 + 15(20) = 750
30	450 + 15(30) = 900
40	450 + 15(40) = 1050
50	450 + 15(50) = 1200
60	450 + 15(60) = 1350



3. Plumbing Elite charges \$100 plus an hourly rate of \$80/h for a home service call. Construct a table of values and a scatter plot to represent the relation between the time required for a service call and the cost.

Study Aid

- For help, see the Review of Essential Skills and Knowledge Appendix.

Question	Appendix
4	A-5 and A-8
6	A-11

PRACTICE

4. Evaluate.

a) $4a + 5$, for $a = 3$

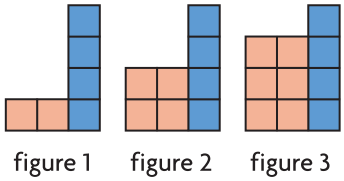
b) $5 - 2b$, for $b = -1$

c) $5p + q$, for $p = \frac{2}{5}$ and $q = \frac{1}{4}$

5. Simplify.

a) $3(x - 2)$ **b)** $0.5(2x - 6)$ **c)** $\frac{1}{4}(16 - 9x)$

6. Consider this pattern.



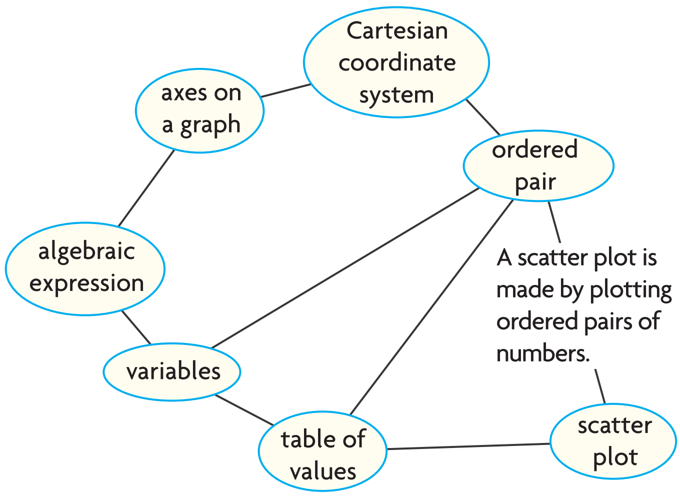
a) Use an algebraic expression to describe the number of tiles in terms of the figure number.

b) How do the colours in the diagram relate to the parts of the algebraic expression?

c) Use the algebraic expression to determine the number of tiles in figure 17.

d) Make a table of values and draw a scatterplot comparing the figure number to the number of tiles in each figure.

7. State how each term is related to each other term. The relation between scatter plot and ordered pair is done for you.



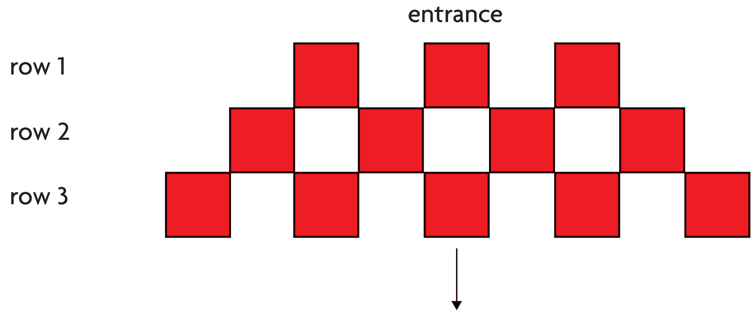
APPLYING What You Know

YOU WILL NEED

- grid paper
- square tiles

Patterning Squares

Marco and Renée are placing red stones in a pattern at the entrance to the skate park. They need to order more red stones to finish the job.



? How many red stones are in the 25th row of this pattern?

A. Determine the number of red stones in row 4, then draw row 5.

B. Copy and complete the table of values.

Row	Number of Red Stones
1	3
2	4
3	
4	
5	

C. Use your table to draw the scatter plot.

D. Write the algebraic expression that relates the number of red stones to the row number.

E. Use the relation in part D to determine the number of red stones in row 20.

F. Determine which row will have 18 red stones.

G. How many red stones are needed for row 25? Explain how you used the pattern rule.

3.1 Relations

YOU WILL NEED

- grid paper

GOAL

Represent a relation using a table of values, a graph, or an equation.

LEARN ABOUT the Math

Chris runs a window-washing service. She charges a flat rate of $5, plus $3 per window.

? How can Chris's customers calculate the cost to wash their windows?

EXAMPLE 1 Representing a relation in different ways

Represent the **relation** between the number of windows washed and the cost to wash them.

relation
a description of how two variables are connected

independent variable
in a relation, the variable whose values you choose; usually placed in the left column in a table of values and on the horizontal axis in a graph

dependent variable
in a relation, the variable whose values you calculate; usually placed in the right column in a table of values and on the vertical axis in a graph

Geri's Solution: Representing the relation with a table of values

Number of Windows	Cost ($)
0	5
1	$3 \times 1 + 5 = 8$
2	$3 \times 2 + 5 = 11$
3	$3 \times 3 + 5 = 14$

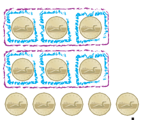
I created a table of values.

The customer chooses the number of windows to wash, so this is the independent variable.

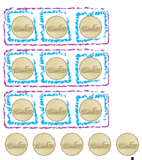
The cost depends on how many windows are washed, so cost is the dependent variable.

Brian's Solution: Representing the relation with a picture

cost to wash 1 window



cost to wash 2 windows



cost to wash 3 windows

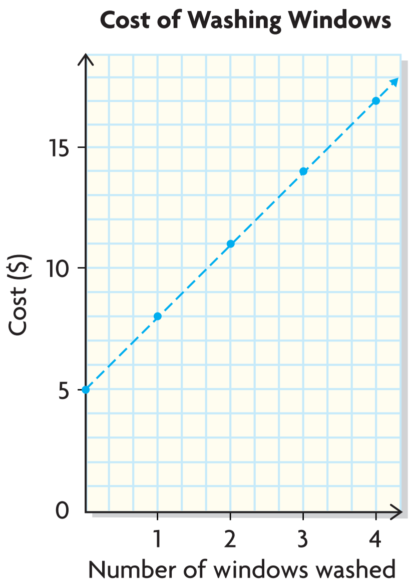
I created a series of pictures.

$5 is constant, so it is the same in each picture.

I circled the part that increases with each additional window washed.

The picture represents the relation between cost and windows washed.

Marlene's Solution: Representing the relation with a graph



I used Geri's table to graph the relation.

The number of windows washed is the independent variable, so I plotted that on the horizontal axis.

Cost is the dependent variable, so I plotted it on the vertical axis.

I connected the points with a dotted line because this set of data is **discrete**. Washing part of a window does not make sense.

discrete
a set of data that cannot be broken into smaller parts

Theo's Solution: Representing the relation with an algebraic expression

Let W represent the number of windows washed.
Let C represent the cost in dollars.

I represented the variables.

Chris charges a flat rate of \$5, plus \$3 per window.

I wrote the relation in words.

cost = (3 × number of windows washed) + 5

Constants go at the end of an equation.

$$C = 3W + 5$$

I replaced the words with variables to create an equation.

Reflecting

A. How do the students' representations all describe the same relation?

B. Which representation would you use? Why?

APPLY the Math

EXAMPLE 2 Solving a problem involving a relation

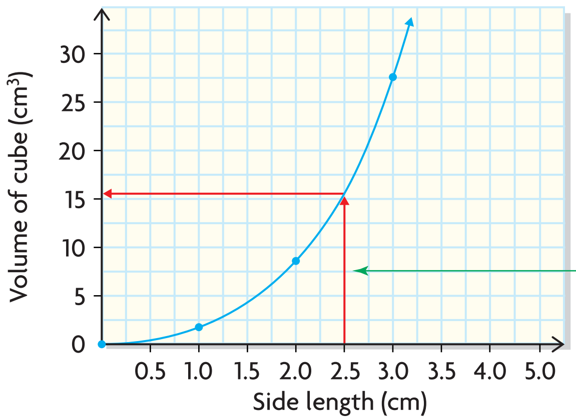
Determine the volume of a cube with a side length of 2.5 cm.

Andrea's Solution: Using a graph to estimate a value

Side Length of Cube (cm)	Volume of Cube (cm^3)
1.0	$1 \times 1 \times 1 = 1^3 = 1$
2.0	$2 \times 2 \times 2 = 2^3 = 8$
3.0	$3 \times 3 \times 3 = 3^3 = 27$

I made a table of side lengths and volumes. I used the side length to calculate volume.

Volume of Cube vs. Side Length



I graphed the relation. This set of data is **continuous**, so I connected the points with a solid line.

I **interpolated**. I drew a line from 2.5 cm on the horizontal axis to the graph.

I drew a line from that point to the vertical axis.

The volume is about 15 cm^3.

My estimate seems reasonable since multiplying $2.5 \times 2.5 \times 2.5$ gives me an exact volume of 15.625 cm^3.

continuous

a set of data that can be broken down into smaller and smaller parts and still have meaning

interpolate

to estimate a value between two known values

Andrea was able to estimate the volume from the graph. Pilar wanted a more exact answer, so she represented the relation using an equation.

Pilar's Solution: Using an equation to determine an exact value

Let x represent the side length of the cube.
Let y represent the volume of the cube.

The volume depends on the side length.

$y = x^3$

$y = 2.5^3$
$y \doteq 15.6$

I described the relation using an equation and calculated the volume for $x = 2.5$.

The volume is 15.6 cm^3.

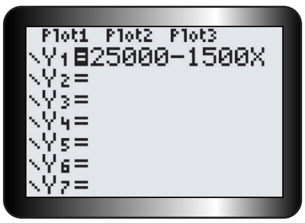
I answered to the nearest tenth, because that is how the side length is given.

You can use a graphing calculator to graph a relation, if you know its equation. You can use this graph to make accurate estimates.

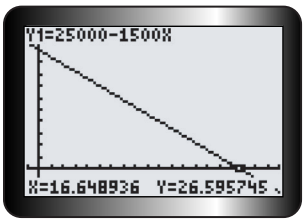
EXAMPLE 3 Using technology to estimate a value

The equation $V = 25\,000 - 1500T$ represents a car's value after T years. When will the car be worth \$0?

Otto's Solution



The independent variable is the car's age. The dependent variable is the car's value.



I graphed the relation and used the TRACE key to estimate when the car would be worth \$0.

The car will be worth \$0 after about 16.7 years.

Tech Support

- See Appendix B-5 for information on how to set a calculator window so that a graph is visible.
- For help using the TRACE key, see Appendix B-4.

In Summary

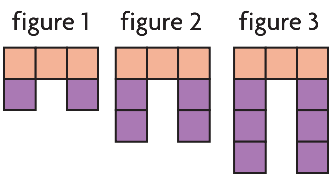
Key Idea

- A relation can be described by a table of values, a graph, an equation, a picture, and words.

Need to Know

- You can use a table of values or a graph to estimate values of a relation.
- You can use an equation to determine exact values of a relation.
- You can graph a relation by entering an equation into a graphing calculator or graphing software.

CHECK Your Understanding



1. Describe a relation between the figure number and the total number of squares using a table of values, a graph, and an equation.

2. Describe each relation using two of the following: a graph, a table of values, a picture, or an equation. Justify your choice.

a) the perimeter of an equilateral triangle in terms of its side length

b) the amount John pays for a taxi ride, if the fare is \$0.50/km plus a flat rate of \$2.50

PRACTISING

3. Graph each relation.

a)

Time (min)	Distance (km)
0	15
5	18
10	21
15	24

b)

Side Length (cm)	Area (cm^2)
1	1
2	4
3	9
4	16

c)

x	y
−2	−4
−1	−2
0	0
1	2

4. Describe each relation in the previous question using an equation.

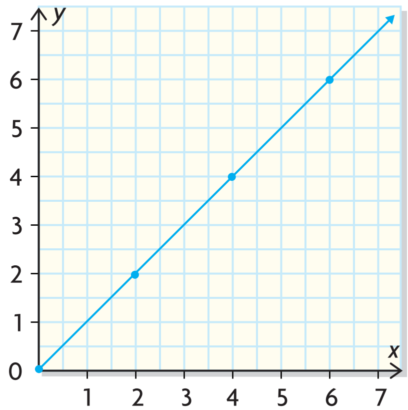
5. Elinor is training for a race. The table shows her times and distances.

Time (min)	Distance (km)
0	0
10	2
20	4
30	6

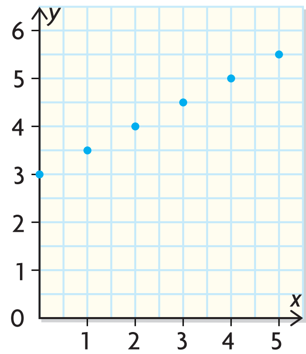
a) Which variable is independent and which is dependent?
b) Estimate the distance Elinor has run after 22 min.
c) Describe the relation using a graph.
d) Verify your estimate in part b).

6. Describe each relation with either a table of values or an equation.

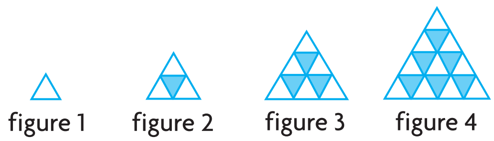
a)



b)



7. This pattern is made of equilateral triangles with sides of 1 cm.



a) Graph the relation between a figure and its perimeter.
b) Determine the perimeter of figure 10. Explain your reasoning.
c) Graph the relation between the figure number and the number of white triangles in the figure.
d) Determine the number of white triangles in figure 10. Explain.

8. The relation between Celsius and Fahrenheit is $C = \frac{5}{9}(F - 32)$.

K **a)** Which variable is independent in this equation? Justify your choice.
b) Describe the relation using a table of values.
c) Graph the relation.
d) Are the data continuous or discrete?
e) Estimate the Celsius temperature when $F = 100$ using your graph.
f) Calculate the Celsius temperature when $F = 100$ using the equation.
g) Why might you predict a value using an equation, instead of a graph or a table?

9. These ordered pairs show the relation between the amount of cell phone use in minutes and the cost, in dollars: (0, 25), (10, 26), (20, 27)
   a) Explain why cost is the dependent variable and what the ordered pair (0, 25) means.
   b) Graph the relation.
   c) Are the data continuous or discrete?
   d) Describe the relation using an equation.
   e) Would you predict the cost of 100 min using a graph, or using an equation? Explain.
   f) Predict the cost of 100 min.

10. Antwan charges \$5/h, plus a flat fee of \$8, in his lawn-mowing business.
   a) Describe the relation between earnings and hours using an equation.
   b) Justify your choice for independent and dependent variables.

11. A van's gas tank holds 75 L. The van uses 0.125 L/km.
   A a) Describe the relation between the distance the van travels and the volume of gas in its tank.
   b) How far can the van travel on a full tank of gas?

12. a) Which of these ordered pairs are points on the graph of $y = 5x$?
   A. (0, 0) **B.** (2, 10) **C.** (4, 15) **D.** (−2, −10)
   b) Which of these ordered pairs are points on the graph of $y = 3x - 6$?
   A. (2, 0) **B.** (5, 9) **C.** (−1, −9) **D.** (6, 10)

13. Represent each relation using a table of values and a graph.
   a) $y = x$
   b) $y = 2x + 3$
   c) $y = -x + 1$
   d) $y = 0.25x - 3.5$
   e) $y = -\frac{1}{2}x$
   f) $y = -\frac{2}{3}x + \frac{1}{6}$

14. Match each table to its graph and equation. Explain your reasoning.

a)

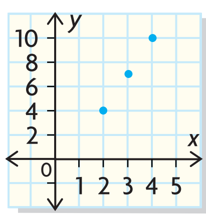
x	y
2	6
3	9
4	12

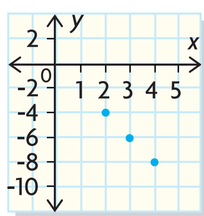
b)

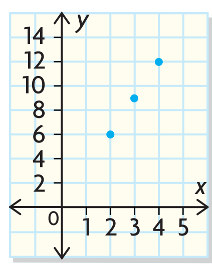
x	y
2	4
3	7
4	10

c)

x	y
2	−4
3	−6
4	−8

A.


B.


C.


i) $y = -2x$
ii) $y = 3x$
iii) $y = 3x - 2$

15. Clarise has \$50 in her piggy bank. She takes \$2.50 from it each week to buy a hot chocolate and a banana from the cafeteria. Create a table of values, a graph, and an equation to describe the amount of money in the piggy bank each week.

16. Jacques surveyed people about their part-time jobs.

Group 1	Group 2
Abe: \$7 per hour (waiting tables)	**Anne:** \$7.50 per hour plus tips of \$25.00
Beth: \$20 per lawn (mowing lawns)	**Boris:** \$12 per lawn plus a flat rate of \$5
Carl: 10% commission (selling furniture)	**Carol:** 7% commission, plus a flat rate of \$50

Suppose Abe and Anne each work for 10 h, Beth and Boris each mow 5 lawns, and Carl and Carol each sell \$1000 worth of goods. Which group earns more?

17. Describe each relation in words.

C **a)** $I = 2.54c$, where I is inches and c is centimetres

b) $F = \frac{9}{5}C + 32$, where F is degrees Fahrenheit and C is degrees Celsius

c) $k = \frac{p}{2.2}$, where p is pounds and k is kilograms

d) $K = C + 273$, where K is degrees Kelvin and C is degrees Celsius

Extending

18. **a)** Graph $y = 2x$, $y = 2x + 2$, $y = 3x$, and $y = 3x - 1$ on the same axes.

b) How do the equations tell you whether the graph will pass through the origin?

19. The table to the right shows several different heights and areas for triangles with a base of 10 cm.

a) Graph the relation between height and area.

b) Write an equation to relate the area of the triangle to its height.

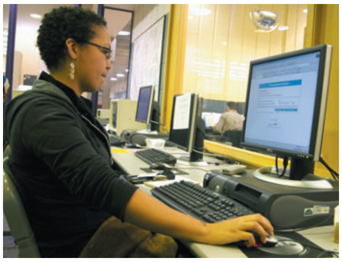
Height (cm)	Area (cm^2)
2	10
5	25
10	50
20	100

20. A rocket's height in metres, h, at time t, in seconds, is given by $h = -5t^2 + 3t + 2$. Describe the relation between height and time with a table of values and a graph.

3.2 Exploring Linear Relations

YOU WILL NEED

- grid paper



GOAL

Identify direct and partial variations.

EXPLORE the Math

Rana's Computer Repair Service charges $45/h. Bill's Computer Repair Service charges a flat fee of $25 plus $18/h. Each company charges for parts of hours.

? How are the plans alike and how are they different?

A. Make a table of values that shows solutions for each company's cost for 0, 1, 2, and 3 hours of service.

B. Graph the relation between cost and hours of service for each company.
How are the graphs alike and how are they different?

C. Use an equation to describe the cost in terms of hours of service, for each company.
How are the equations alike and how are they different?

D. Identify the ***y*-intercept** of each relation. What does it mean in each case?

E. How are the hourly rate and initial value connected to the table of values, graph, and equation for each relation?

F. Identify each relation as a **direct variation** or a **partial variation**. Justify your answer.

Reflecting

G. Why might you have predicted that each graph would be a **linear relation**?

H. If the service time triples, Rana's charge will triple but Bill's won't. Why is that so?

I. How are direct and partial variations alike and how are they different? Refer to graphs, tables, and equations.

***y*-intercept**
the value of the dependent variable when the independent variable is zero; sometimes called the initial value

direct variation
a relation in which one variable is a multiple of the other

partial variation
a relation in which one variable is a multiple of the other plus a constant amount

linear relation
a relation in which the graph forms a straight line

In Summary

Key Idea

- You can determine whether a linear relation is a partial or a direct variation by examining its table of values, its graph, or its equation.

Direct Variation	Partial Variation
(0, 0) is an ordered pair in the table of values.	(0, 0) is not an ordered pair in the table of values.
The initial value is 0, so the graph passes through (0, 0).	The initial value is some number, b, so the graph passes through (0, b).
The equation looks like $y = mx$.	The equation looks like $y = mx + b$.

Need to Know

- A solution to a linear relation is an ordered pair that appears in the table of values, lies on the line representing the linear relation, or makes a true statement in the equation of the relation.
- An initial value has a corresponding *x*-value of zero.

FURTHER Your Understanding

1. Identify each relation as a direct or a partial variation. Support your answer using a table, a graph, and form of the equation.

a) $y = 2x$ **c)** $y = 1 - x$ **e)** $y = -\frac{1}{2}x$

b) $y = 2x + 3$ **d)** $y = 0.25x - 3.5$ **f)** $y = -\frac{2}{3}x + \frac{1}{6}$

2. A small rocket is launched from a hill 1500 m above sea level. It rises at 35 m/s.

a) Write an equation for the relation between the height of the rocket and time.

b) Use a table of values to graph this relation.

c) Identify this relation as a direct or a partial variation. Explain.

3. Students can choose from two different cafeteria milk plans.
Plan A: Pay \$0.75 per glass of milk
Plan B: Pay \$10, plus \$0.25 per glass of milk

a) Write an equation for each plan.

b) Determine the cost of 20 glasses for each plan.

c) Determine the cost of 30 glasses for each plan.

d) Which plan would you choose? Why?

e) Identify each plan as a direct or a partial variation.

f) How does the type of variation affect the cost?

3.3 Investigating Properties of Linear Relations

YOU WILL NEED

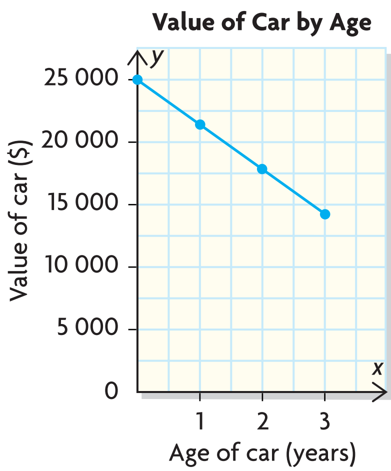
- grid paper

GOAL

Identify properties of linear relations.

INVESTIGATE the Math

Cole bought a new car for $25 000. This graph shows its value over the first three years.



When will Cole's car be worth $0?

A. Calculate the amount by which Cole's car decreased in value between years 1 and 2.

B. Calculate the **rate of change** in the car's value between years 1 and 3.

C. Calculate the **slope** of the graph between years 1 and 3.

How does the slope compare to your answer in part A?

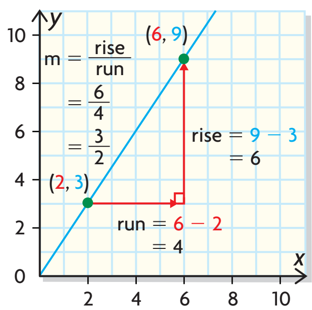
D. Copy the following table. Complete the **first difference** column. How do the first differences compare to the slope in part C?

Age of Car, x	Value of Car, y	First Difference, Δy
0	25 000	
		21 425 − 25 000 = ■
1	21 425	
		17 850 − 21 425 = ■
2	17 850	
3	14 275	

rate of change
the change in one variable relative to the change in another

slope
a measure, often represented by m, of the steepness of a line; the ratio comparing the vertical and horizontal distances (called the rise and run) between two points;

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$



first difference
the difference between two consecutive y-values in a table in which the difference between the x-values is constant

E. Copy and complete the following table.
Why are the first differences different than in part D?

Age of Car, x	Value of Car, y	First Difference, Δy
0	25 000	
		17 850 − 25 500 = ■
2	17 850	
		■ − 17 850 = ■
4	■	
		■ − ■ = ■
6	■	

Communication Tip

- The Greek letter "Δ" (delta) represents change, so Δy represent the difference between two y-values.

F. Write an equation for the relation between the car's value and its age. Which parts represent the first differences, the slope, and the y-intercept?

G. Determine the ***x*-intercept** of the graph. Use it to tell when Cole's car will be worth \$0. How do you know?

***x*-intercept**
the value at which a graph meets the x-axis; the value of y is 0 for all x-intercepts

Reflecting

H. What is the connection between the first differences and the slope?

I. When you calculated the slope, did it matter which points you chose? Explain.

J. Use the graph to explain why the first differences were constant.

APPLY the Math

EXAMPLE 1 Applying the connection between slope and rate of change

Andrea and Dana had cycled 30 km after two hours and 60 km after four hours. At what rate were they cycling?

Dana's Solution: Using a table of values

Time in Hours, x	Distance in Kilometres, y	First Difference, Δy
0	0	
		30 − 0 = 30
2	30	
		60 − 30 = 30
4	60	

I made a table of values and calculated the first differences.

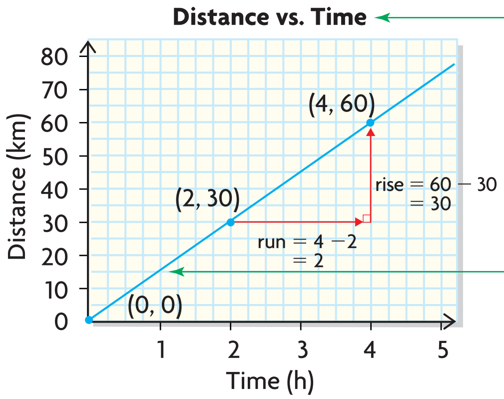
The x-values increased by 2, so $\Delta x = 2$.

$$\text{rate of change} = \frac{\Delta y}{\Delta x} = \frac{30 \text{ km}}{2 \text{ h}} = 15 \text{ km/h}$$

I calculated the rate of change.

We were cycling at a rate of 15 km/h.

Andrea's Solution: Using a graph



The distance depended on time, so I chose time as the independent variable.

I plotted the points and drew a line through them. I assumed they were cycling at a constant rate. I used a solid line because the data are continuous.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{30 \text{ km}}{2 \text{ h}} = 15 \text{ km/h}$$

I calculated the slope of the line.

We were cycling at a rate of 15 km/h.

The slope gives the speed or rate they were cycling.

Communication *Tip*

- We call the horizontal distance between two points the run. The run is positive when it goes to the right and negative when it goes to the left. We call the vertical distance between two points the rise, even if the line goes downward. The rise is positive when it goes upward and negative when it goes downward.

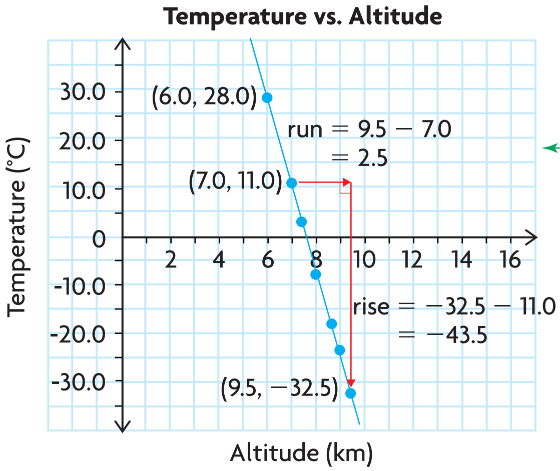
When the data for the independent variable in a table of values do not increase by an equal amount, graphing the data can help you determine if the relationship is linear. If it is, calculating the slope of the line will give you the rate of change of the dependent variable.

EXAMPLE 2 Using a graphing strategy to estimate rate of change

A weather balloon recorded the temperature at these altitudes. Estimate the rate of change of the temperature.

Altitude (km)	6.0	7.0	7.6	8.1	8.7	9.0	9.5
Temperature (°C)	28.0	11.0	2.5	−7.7	−17.9	−23.0	−32.5

Martha's Solution



I didn't use first differences because the altitude was recorded at irregular times.

I graphed the points and connected them with a solid line.

The graph is straight, so this relation is linear.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-43.5\ °\text{C}}{2.5\ \text{km}}$$
$$= -17.4\ °\text{C/km}$$

I calculated the slope, because it has the same value as the rate of change.

The rate of change is $-17.4\ °\text{C/km}$.

When the altitude increases by 1 km, the temperature decreases by 17.4 °C.

In Summary

Key Idea

- To determine the rate of change of a linear relation, you can do the following:
  - Calculate the first differences in a table in which the *x*-values increase or decrease by 1.
  - Calculate the slope, $\frac{\text{rise}}{\text{run}}$, using any two points on a graph of the relation. The rate of change has the same value as the slope.

Need to Know

- If the independent values in a table change by a constant amount other than 1, the ratio of the first differences to the change in *x*, $\frac{\Delta y}{\Delta x}$, is the slope, or the rate of change.
- Δy is the change in *y* and is equivalent to the rise.
  Δx is the change in *x* and is equivalent to the run.

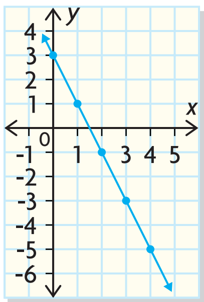
CHECK Your Understanding

1. Which of these relations are linear? How do you know?

a)

x	y
1	3
2	6
3	9
4	12

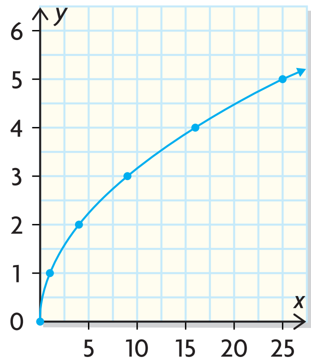
b)



c)

x	y
1	1
2	4
3	9
4	16

d)



2. Determine the rate of change for each linear relation in question 1.

PRACTISING

3. Determine the rate of change in each linear relation.

a)

x	y
2	11
4	17
6	23
8	29
10	35

b)

x	y
5	0
4	2
3	4
2	6
1	8

c)

x	y
0	0
0.25	2
0.5	4
0.75	6
1	8

d)

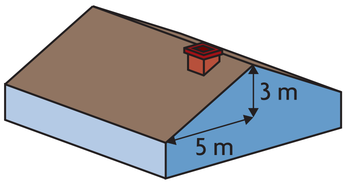
x	y
1	−2
4	−8
5	−10
3	−6
2	−4

4. a) Why might this table mislead you about whether the relation is linear?

Age of Car in Years, *x*	0	3	1	2
Value of Car in Dollars, *y*	15 000	6 000	12 000	9 000

b) Graph the data.
c) What is the slope and what does it mean?
d) What is the *x*-intercept? Does this seem realistic? Explain.

5. What is the slope of this roof?

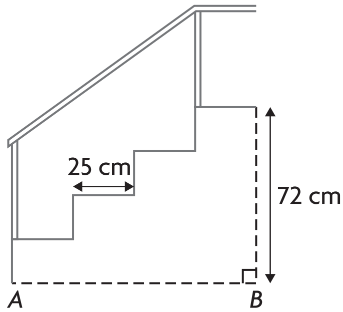


6. There are three steps from the ground to a front porch 72 cm above the ground, as shown.

a) What is the rise of each step?

b) The horizontal distance across each step is 25 cm. Determine the length of *AB*.

c) Determine the slope of the handrail.



7. Determine the slope of the line that passes through each pair of points.

K **a)** (3, 5) and (0, 2)

b) (3, 3) and (−2, 2)

c) (21, −10) and (20, 24)

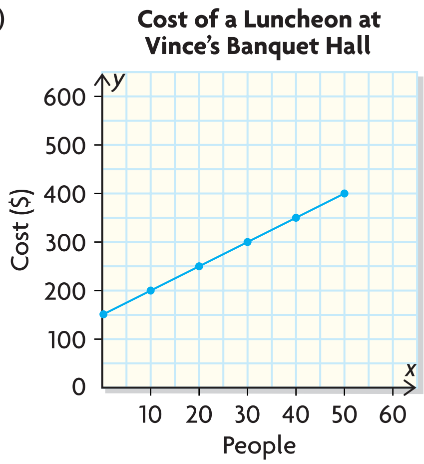
d) (4, 0) and (6, 18)

e) (1, −1) and (2, 2)

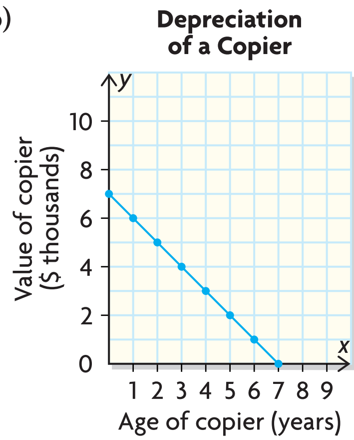
f) (−3, −8) and (−5, −6)

8. Use the title and axis labels of each graph to tell what the *y*-intercept and slope mean in each case.

a)



b)



9. Determine two more ordered pairs for each relation. Explain your reasoning.

a) rise is 2, run is 3; (2, 5) lies on the line

b) rise is −3, run is 4; (0, −2) lies on the line

c) rise is 5, run is 1; (1, −6) lies on the line

d) rise is −2, run is 1; (−2, −3) lies on the line

Altitude (km)	Air Pressure (Pa)
1	80 000
3	60 000
6	40 000
16	20 000
22	10 000
30	5 000

10. **a)** Graph the data in the table to the left.

A **b)** How does the graph show the rate of change?

c) Estimate the air pressure at an altitude of 20 km.

11. Graph each relation and state the slope.

a) $y = 3x$ **c)** $y = \frac{3}{4}x + 1$ **e)** $y = -x$

b) $y = -2x$ **d)** $y = -\frac{1}{5}x + 1$ **f)** $y = \frac{2}{3}x - 4$

12. An equation for a house's value is $y = 7500x + 125\,000$, where y is the value in dollars and x is the time in years, starting now.

a) What is the current value of the house?

b) What is the value of the house 2 years from now?

c) Determine the value of the house in 7 years.

d) At what rate is the house value changing from year to year?

13. The amount of money in Alexander's account is $y = 4000 - 70x$, where y is the amount in dollars and x is the time in weeks.

a) Which variable is independent and which is dependent?

b) How do you know the relation is linear?

c) Determine the rate of change of the money in Alexander's account.

d) What does the rate of change mean?

e) How does the rate of change relate to the equation?

f) When will Alexander's account be empty?

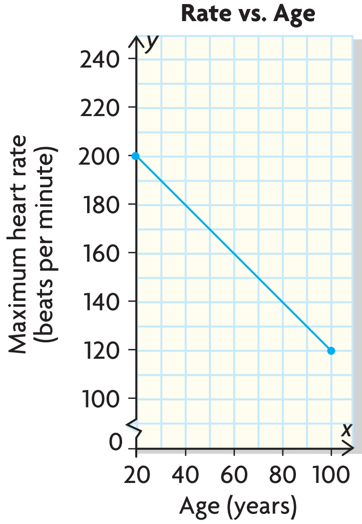
14. This graph shows the maximum heart rate a person should try to achieve while exercising.

a) What does the y-intercept mean?

b) What does the slope represent?

c) Write an equation for the line.

d) Estimate the maximum heart rate for a 58-year-old.



15. Jae-Ho works at a clothing store. He earns a weekly salary of \$300 and **T** 5% commission on his total weekly sales. He thinks that if his commission doubles, so will his earnings. Is he right? Justify your answer.

16. Marie earns \$1 for every 4 papers she delivers.
C

a) Show that the relation between papers delivered and money earned is linear, using a graph and a table of values.
b) What do the first differences mean?
c) What is the rate of change of Marie's earnings?
d) Predict Marie's earnings for delivering 275 papers using an equation.

Extending

17. Write a linear equation for each line.

a) a line with a y-intercept of 2 and a slope of $\frac{3}{5}$
b) a line with a y-intercept of 0 and a slope of -4
c) a line that passes through (0, 5) and (3, 6)
d) a line with a slope of 2 that passes through (4, 1)
e) a line with a slope of $-\frac{1}{2}$ that passes through (3, 0)
f) a line with a slope of $\frac{4}{7}$ that passes through (7, 2)

18. **a)** Make a table of values and a graph for each relation.
C **b)** Copy and complete the table using your results from part a).
c) What connections do you see between each equation and the values in the table?

Equation	Slope	y-intercept	x-intercept
$2x + 5y = 10$			
$4x - 2y = 7$			
$x + y = -2$			

3 Mid-Chapter Review

FREQUENTLY ASKED Questions

Study Aid

- See Lesson 3.1, Examples 1, 2, and 3.
- Try Mid-Chapter Review Question 1.

Q: How can you describe a relation between two variables?

A: You can use a picture, a table of values, a graph, or an algebraic equation.

EXAMPLE

Tickets to an amusement park cost \$6, plus \$2 per ride. Describe the relation between the number of rides and the cost.

Solution

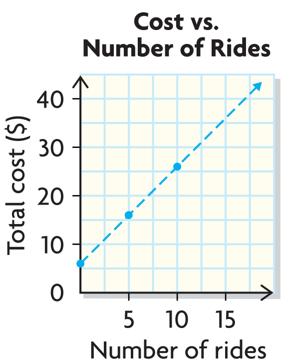
\$6 is constant, so it is the same in each picture.
The picture shows the cost increases by \$2 with each additional ride.

Solution

Number of Rides	Cost
0	6
1	8
2	10
3	12

The table shows the cost increases by \$2 with each additional ride.

Solution



Calculate the cost for 0 rides, 5 rides, and 10 rides. Use the number of rides as the independent variable and the cost as the dependent variable. Only whole numbers of rides make sense, so the data are discrete. Connect the points with a dotted line.

Solution

Let R represent the number of rides, the independent variable.
Let C represent the cost in dollars, the dependent variable.
Cost $= 2 \times$ number of rides $+ 6$
$C = 2R + 6$

Q: How do you know whether a linear relation is a direct variation or a partial variation?

A: You can use a table of values or a graph.

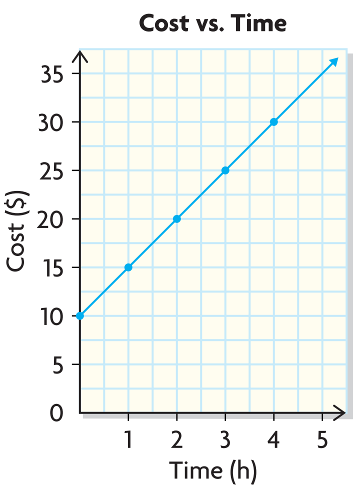
EXAMPLE

Mark delivers groceries to senior citizens for a flat fee of $10 plus $5 per hour and he charges for part hours. Identify the relation between cost and time as a direct or partial variation.

Solution

Time (h)	Cost ($)
0	10
1	15
2	20
3	25
4	30

Each cost is calculated as $5 \times h + 10$. That means it is a partial variation.



The graph is a straight line, so the relation is linear. The graph passes through (0, 10), so it is a partial variation.

Study Aid

- See Lesson 3.2, In Summary.
- Try Mid-Chapter Review Question 2.

Study *Aid*

- See Lesson 3.3, Examples 1 and 2.
- Try Mid-Chapter Review Questions 3, 4, and 5.

Q: How can you determine the rate of change of a linear relation?

A: You can use a table of values or a graph to calculate slope.

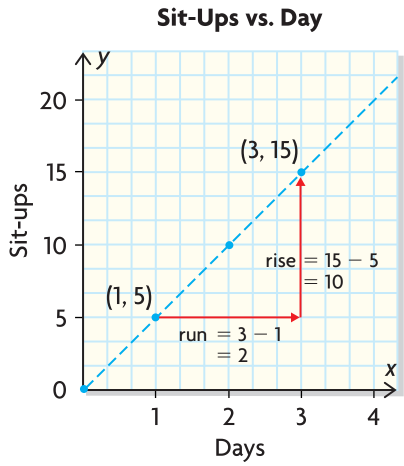
EXAMPLE

Janelle did 5 sit-ups on day 1, 10 sit-ups on day 2, and 15 sit-ups on day 3. Determine the rate of change between sit-ups and days.

Solution

Day, x	Sit-Ups, y	First Difference, Δy
0	0	
		$5 - 0 = 5$
1	5	
		$10 - 5 = 5$
2	10	
		$15 - 10 = 5$
3	15	

The x-variables increase by 1, so the first differences give the rate of change. The rate of change is 5 sit-ups per day.



The set of data is discrete, so connect the points with a dotted line. Choose two points and calculate the slope.

$$\begin{aligned} \text{slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{10 \text{ sit-ups}}{2 \text{ days}} \\ &= 5 \text{ sit-ups/day} \end{aligned}$$

The rate of change is 5 sit-ups per day.

PRACTICE Questions

Lesson 3.1

1. a) Use a table of values, a graph, and an algebraic expression to describe the relation between the number of circles in each figure and the number of stars in the figure.

figure 1 figure 2 figure 3

b) How many circles would be in figure 75?

c) How many stars would be in figure 75?

Lesson 3.2

2. Identify each relation as a partial or a direct variation. Justify your answer.

a) A hockey player is paid \$2 million per year, plus a signing bonus of \$500 000.

b) Cheryl has \$800 in her bank account. She adds \$25 to the account each week.

c) Anthony bicycles at 5 m/s.

Lesson 3.3

3. Determine the rate of change in each relation.

a)

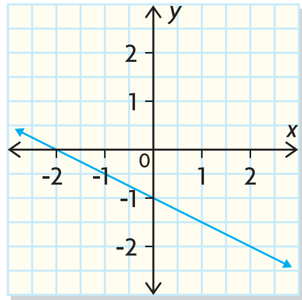
x	*y*
3	1
6	11
9	21

b) The number of words Ming can type is related to time. He can type 200 words in 5 min.

c) $y = 3x$

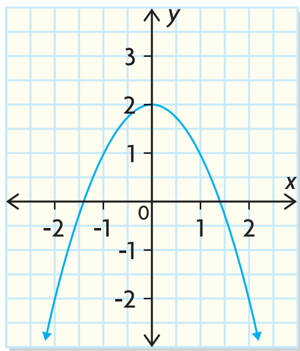
4. Is each relation linear? Explain your reasoning.

a)



b) $y = 0.25x - 3$

c)

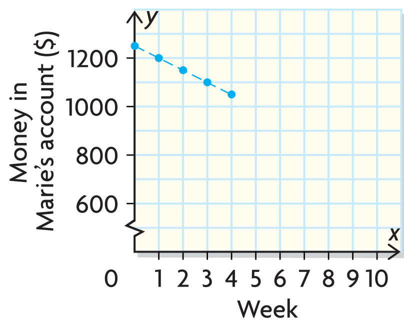


d)

x	−3	4	−1
y	−6	8	−2

5. Marie withdraws the same amount from her account each week, as shown.

Money in Marie's Account by Week



a) What is the slope and what does it mean?

b) What is the *y*-intercept and what does it mean?

c) When will Marie's account be empty?

3.4 Equivalent Linear Relations

GOAL

Represent a linear relation in a different form.

LEARN ABOUT the Math

A health food store is making a mix of nuts and raisins. Nuts are \$30/kg and raisins are \$10/kg. The total mix should cost \$150. $30n + 10r = 150$ describes the relation between the mass of nuts, n, and the mass of raisins, r, that cost \$150.

? What combinations of nuts and raisins cost \$150?

EXAMPLE 1 Connecting the equation and the graph

Determine the combinations of nuts and raisins that will cost \$150.

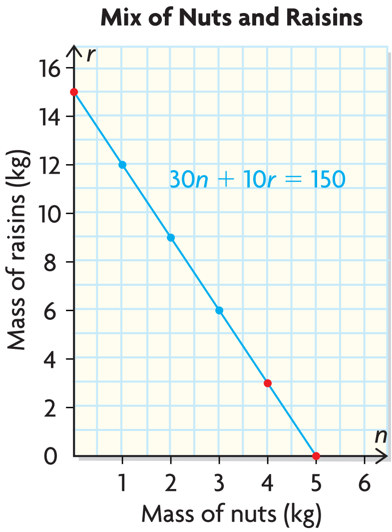
Chelsea's Solution: Using a graphing strategy to determine values of a relation

Mass of Nuts (kg)	Mass of Raisins (kg)
1	$30(1) + 10r = 150$ $30 + 10r = 150$ $30 + 10r - 30 = 150 - 30$ $10r = 120$ $\frac{10r}{10} = \frac{120}{10}$ $r = 12$
2	$30(2) + 10r = 150$ $60 + 10r = 150$ $60 + 10r - 60 = 150 - 60$ $10r = 90$ $\frac{10r}{10} = \frac{90}{10}$ $r = 9$
3	$30(3) + 10r = 150$ $90 + 10r = 150$ $90 + 10r - 90 = 150 - 90$ $10r = 60$ $\frac{10r}{10} = \frac{60}{10}$ $r = 6$

I calculated the mass of raisins needed for 1 kg, 2 kg, and 3 kg of nuts.

Mass of Nuts (kg)	Mass of Raisins (kg)	First Differences
1	12	
		−3
2	9	
		−3
3	6	

The first differences were the same, so the relation is linear.



I used nuts as the independent variable.

I drew a solid line, since the data are continuous.

I determined other combinations by identifying other points on the line.

I only used positive numbers for the masses, since it doesn't make sense to talk about a negative mass.

Some possibilities are no nuts and 15 kg of raisins, 4 kg of nuts and 3 kg of raisins, and 5 kg of nuts and no raisins.

Bob used the intercepts to graph the relation quickly.

Bob's Solution: Using the intercepts as a strategy to graph the relation

$30n + 10r = 150$

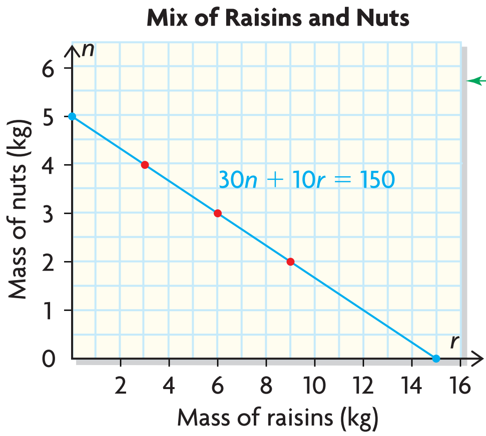
$30y + 10x = 150$

The equation is of degree 1, so the relation is linear. I chose raisins as the independent variable. I replaced *r* with *x* and *n* with *y*.

x-intercept	*y*-intercept
$y = 0$	$x = 0$
$30(0) + 10x = 150$	$30y + 10(0) = 150$
$10x = 150$	$30y = 150$
$\frac{10x}{10} = \frac{150}{10}$	$\frac{30y}{30} = \frac{150}{30}$
$x = 15$	$y = 5$

I used the *x*- and *y*-intercepts to draw the graph.

Two mixes that cost $150 are 15 kg of raisins and no nuts and 5 kg of nuts and no raisins.



The x- and y-intercepts indicate where the mix has just one ingredient.

I drew a solid line, since the data are continuous.

I could interpolate many combinations because both whole number and decimal values make sense.

Some combinations are 3 kg of raisins and 4 kg of nuts, 6 kg of raisins and 3 kg of nuts, and 9 kg of raisins and 2 kg of nuts.

Reflecting

A. Chelsea and Bob chose different independent and dependent variables. How are their tables of values and graphs alike and how are they different?

B. Why did it not matter which variable was independent?

C. How did both Chelsea and Bob show that the relation is linear?

D. Why did Chelsea and Bob create graphs using only the first quadrant?

APPLY the Math

EXAMPLE 2 Using intercepts as a strategy to graph a line

Graph the line $2x + \frac{3}{4}y = 21$.

Greg's Solution

$$2x + \frac{3}{4}y = 21$$

$$2(0) + \frac{3}{4}y = 21$$

I set $x = 0$ to determine the y-intercept.

$$0 + \frac{3}{4}y = 21$$

$$y = 21 \div \frac{3}{4}$$

$$y = 21 \times \frac{4}{3}$$

$$y = 28$$

I solved the equation.

The y-intercept is 28.

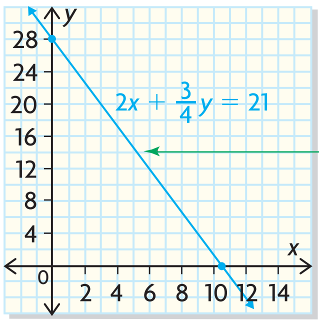
$$2x + \frac{3}{4}(0) = 21$$

I set $y = 0$ to determine the x-intercept.

$$2x = 21$$
$$x = 10.5$$

I solved the equation.

The x-intercept is 10.5.



The equation is of degree 1, so the relation was linear. I drew a line through (0, 28) and (10.5, 0).

Since the variables x and y can be any number, I put arrows on the end of the line to show this relationship continues in both directions.

This is the graph of $2x + \frac{3}{4}y = 21$.

EXAMPLE 3 Using a table of values to draw a graph

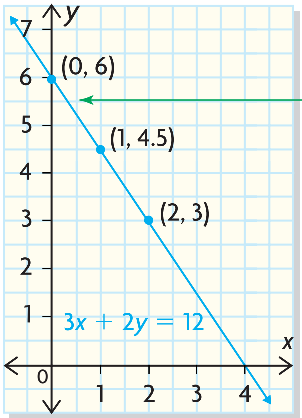
Complete the table of values for $3x + 2y - 12 = 0$ and then graph the line.

x	0	1	2
y			

Claire's Solution

x	y
0	$3(0) + 2y - 12 = 0$ $2y - 12 = 0$ $y = 6$
1	$3(1) + 2y - 12 = 0$ $2y = 0 - 3 + 12$ $2y \div 2 = 9 \div 2$ $y = 4.5$
2	$3(2) + 2y - 12 = 0$ $6 + 2y - 12 = 0$ $2y = 0 - 6 + 12$ $y = 3$

I solved for y for each value of x.



I plotted (0, 6), (1, 4.5), and (2, 3) and drew a solid line through them. I knew the graph was linear because the equation of degree 1.

Since the variables x and y can be any number, I put arrows on the end of the line to show this relationship continues in both directions.

This is the graph of $3x + 2y = 12$.

A graph of a relation can often be a useful tool to help you solve problems.

EXAMPLE 4 Using intercepts to solve a problem

Mia is travelling to a campsite 52 km away. She plans to bike part of the way and canoe the rest of the way. She can bike at 13 km/h and paddle her canoe downstream at 8 km/h. Determine three combinations of biking and canoeing distances Mia can use.

Mia's Solution

If x represents the time in hours I bike, then $13x$ is the distance I can bike.
If y represents the time in hours I canoe, then $8y$ is the distance I can canoe.

The total distance is the distance biking plus the distance paddling.

It doesn't matter which variable is independent.

$$\begin{aligned} \text{Total distance} &= 13x + 8y \\ 52 &= 13x + 8y \end{aligned}$$

The equation is of degree 1, so the relation is linear.

$$\begin{aligned} 52 &= 13(0) + 8y \\ 52 &= 8y \\ 52 \div 8 &= 8y \div 8 \\ 6.5 &= y \end{aligned}$$

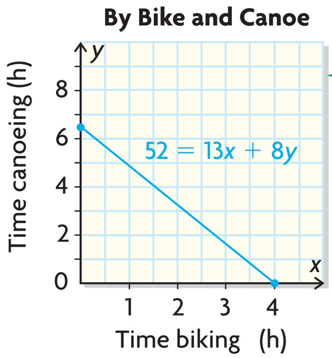
I calculated the y-intercept by setting $x = 0$.

I can canoe for 6.5 h and bike for 0 h.

$$\begin{aligned} 52 &= 13x + 8(0) \\ \frac{52}{13} &= \frac{13x}{13} \\ 4 &= x \end{aligned}$$

I calculated the x-intercept by setting $y = 0$.

I can bike for 4 h and canoe for 0 h.



I graphed the relation using the x- and y-intercepts. I identified points on the line and how far I can bike and canoe to reach my campsite.

I drew the graph only in the first quadrant since the time canoeing or biking must be 0 or greater.

I can bike for 1 h and canoe for just under 5 h or any other (x, y) combination on the line.

In Summary

Key Ideas

- You can write a linear relation in the forms $Ax + By = C$ or $Ax + By + C = 0$, or in the form $y = mx + b$.
- You can use the form $Ax + By = C$ to determine the x- and y-intercepts and use them to graph a linear relation.

Need to Know

- The x-intercept is the point at which the line meets or crosses the x-axis. The coordinates of the x-intercept are $(x, 0)$. To determine the x-intercept, set $y = 0$ and solve for x.
- When you write a linear relation in the form $Ax + By = C$, usually it does not matter which variable is independent.

CHECK Your Understanding

1. Graph each relation using the x- and y-intercepts.

a) $-3x + 2y = 6$ **b)** $\frac{1}{2}x + \frac{2}{3}y = \frac{1}{6}$ **c)** $y = 2x - 1$

2. Marie works at a boutique and at a travel agency. In all, she works for 38 h per week.

a) Write a linear relation to model this case. Use x for the number of hours she works at the boutique, and y for the number of hours she works at the travel agency.

b) What do the x- and y-intercepts mean?

PRACTISING

3. Locate three points on the line $6x - y = 18$, where x and y are both integers, and draw the line.

4. Graph each relation using the x- and y-intercepts.
   a) $2x - 5y = 10$
   b) $4x + 5y = 20$
   c) $x + y = 0$
   d) $2x + 3y = 0$

5. Nicolas has \$14.50 in quarters and dimes.
   a) Explain why $0.10x + 0.25y = 14.50$ models this case.
   b) Is this relation linear? Explain.
   c) Determine the x- and y-intercepts. What do they mean?
   d) Are the data continuous or discrete? Explain.

6. Amir earns \$9/h working in a coffee shop and \$11.25/h working in a grocery store. Last week he earned \$288.
   a) Explain why $9x + 11.25y = 288$ models this case.
   b) Is this relation a straight line? Explain.
   c) Determine the x- and y-intercepts. What do they mean?
   d) For how many hours might Amir have worked in each place?

7. **K** A boat travels down the St. Lawrence River at 30 km/h and moors at a spot where the passengers can watch whales. After a while, it travels back up the river to its starting point at 20 km/h. The boat travels 60 km in all.
   a) Explain why $30x + 20y = 60$ models this problem. Explain what x and y represent.
   b) Determine the x- and y-intercepts. What do they mean?
   c) Graph the relation.

8. Graph each relation using the x- and y-intercepts.
   a) $2x + \frac{4}{5}y = 11$
   b) $\frac{1}{4}x - \frac{2}{3}y = 1$
   c) $-\frac{5}{6}x + y = -\frac{1}{3}$
   d) $-\frac{1}{8}x + \frac{2}{5}y = \frac{2}{25}$

9. **T** Two airplanes appear on the same radar screen with a coordinate grid. The path of one plane is $y = \frac{2}{5}x - 2$ and the path of the other is $2x - 5y - 7 = 0$. Do the paths cross?

10. **A** Henri charges \$3 to sharpen a pair of figure skates and \$2.50 to sharpen a pair of hockey skates. Last Sunday, he earned \$240.
    a) Determine two possible numbers of pairs of figure skates and hockey skates that Henri could have sharpened.
    b) Henri sharpened 94 pairs of skates. How many of each type did he sharpen to earn \$240?

11. Julia is preparing a mix of raisins and nuts for her brother's party. Chocolate raisins are \$0.88/100 g and peanuts are \$1.00/100 g. She plans to spend \$3.00 on the mix.

a) Explain why $0.88x + 1y = 3$ models this case.

b) Determine the x- and y-intercepts. What do they mean?

c) Graph the relation.

12. Jennifer decides to invest \$1200 in mutual funds. One stock is \$3.50/share and the other is \$5.75/share. How many shares of each stock can she buy? Give two possible answers.

13. Copy and complete the table.

C

	Relation	x-intercept	y-intercept	Slope
a)	$2x + 3y = 6$			
b)	$x + 4y = 9$			
c)	$-2x + 5y = -10$			
d)	$4x - 6y = 8$			

Extending

14. Look at your answer to question 13. What patterns can help you to determine the slope of any line in the form $Ax + By = C$?

15. Write each equation in $y = mx + b$ form.

a) $4x + 2y = 8$

b) $x + y = 2$

c) $3x - 4y + 1 = 0$

16. Write the equation of each line in the form $Ax + By = C$.

a) x-intercept 4, y-intercept -2

b) x-intercept $\frac{1}{2}$, y-intercept $\frac{3}{4}$

c) x-intercept 1, y-intercept -3

d) x-intercept $\frac{2}{3}$, y-intercept 3

17. The relation $y = 2x^2 - 8$ is not linear. It has two x-intercepts and one y-intercept.

a) What is the y-coordinate for both x-intercepts?

b) Calculate the x-intercepts.

c) Calculate the y-intercept.

d) Create a table of values and graph $y = 2x^2 - 8$ to verify your answers to parts b) and c).

Curious | Math

Linear Relations in the Human Body

Archaeologists use linear relations to determine an adult human's height. The relations they use differ depending on the subject's ethnicity. Also, the relations used are more reliable for predicting the heights of adults than they are for children since an adult's height does not change as much as a child's height does.

For example, the height of an adult female is $H = 2.50t + 74.70$, where H is the female's height and t is the length of the tibia (largest lower leg bone, from knee to ankle), both in centimetres.

The height of an adult male is $H = 2.38t + 78.8$.

1. Test the formulas on yourself and your classmates.
2. Graph the relation between height and tibia length for females and for males.
3. Leonardo da Vinci discovered similar linear relations. For example, the average adult human figure is 7 heads high.

   Write this relation and use it to predict your height and that of your classmates.
4. Measure the height and width of your head.

   Predict the relation between head width and shoulder width.

   Test your prediction on yourself and your classmates.
5. Measure the distance from the end of your wrist to the end of your fingers. Predict the relation between head height and hand length.

   Test your prediction on yourself and your classmates.
6. Can you find other linear relations in the human body?

   Investigate and test your hypotheses.

3.5 Linear and Nonlinear Relations

GOAL

Recognize whether a relation is linear or nonlinear.

YOU WILL NEED

- grid paper

LEARN ABOUT the Math

Mario is playing a video game in which you gain extra lives by capturing pots of gold. Mario can choose one of two options. He thinks he can capture at least six pots of gold.

Option 1	
Pots of Gold	**Lives Gained**
1	5
2	10
3	15
4	20

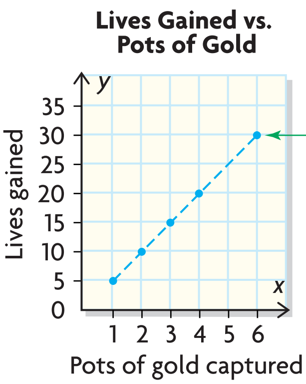
Option 2	
Pots of Gold	**Lives Gained**
1	1
2	4
3	9
4	16

? Which option should Mario choose?

EXAMPLE 1 Applying properties of linear and nonlinear relations

Determine which option Mario should choose to gain the most extra lives.

Mario's Solution: Thinking about graphs



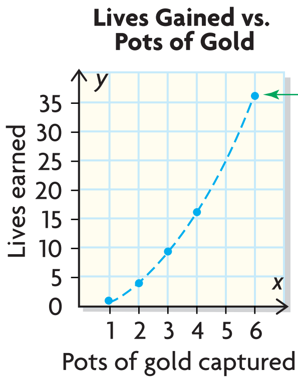
I graphed Option 1. The points lay on a straight line, so it is a linear relation.

I extrapolated. Option 1 gives me 30 lives for six pots.

extrapolate
to predict a value by following a pattern beyond known values

nonlinear relation

a relation whose graph is not a straight line



I graphed Option 2. The points did not lie on a straight line, so it is a nonlinear relation. I extrapolated. Option 2 gives about 35 or 36 lives for six pots.

I'll choose Option 2 because it gives me more lives.

You can also determine whether a relation is linear or nonlinear from a table of values.

Mika's Solution: Thinking about tables of values

Option 1

Pots of Gold	Lives Gained	Δy
1	5	
		5
2	10	
		5
3	15	
		5
4	20	

The first differences in Option 1 are constant, so this relation is linear.

Option 2

Pots of Gold	Lives Gained	Δy
1	1	
		3
2	4	
		5
3	9	
		7
4	16	
		9
5	25	
		11
6	36	

The first differences in Option 2 are not constant, so this relation is nonlinear.

Mario should choose Option 2.

Mario gets 30 lives with Option 1 and 36 lives with Option 2.

You can also decide if a relation is linear or not from the degree of its equation.

Louisa's Solution: Thinking about the equations

Let p represent the number of pots of gold.
Let L represent the number of lives gained.

I chose pots of gold as the independent variable and lives gained as the dependent variable.

Option 1: $\text{lives} = 5 \times \text{pots of gold}$

$$L = 5p$$
$$L = 5(6)$$
$$= 30$$

Option 1 gives five lives for each pot.

I created an equation for Option 1. It is of degree 1, so the relation is linear.

I solved for L when $p = 6$. With Option 1, Mario gets 30 lives.

Option 2: $\text{lives} = (\text{pots of gold})^2$

$$L = p^2$$
$$L = 6^2$$
$$= 36$$

I created an equation for Option 2. It is of degree 2, so the relation is nonlinear.

I solved for $p = 6$. With Option 2, Mario gets 36 lives.

Mario should choose Option 2, because it gives more lives.

Reflecting

A. Why would you expect the first differences to be constant for a linear relation but not constant for a nonlinear relation?

B. How can you tell from a table, a graph, and an equation if a relation is linear or nonlinear?

APPLY the Math

EXAMPLE 2 Using an algebraic strategy to identify a linear relation

The circumference of a circle is the diameter multiplied by π. Identify the relation between circumference and diameter as linear or nonlinear.

Kee's Solution

Let d represent the diameter and C represent the circumference.
Then, $C = \pi d$.

I created an equation for the relation.

The equation is of degree 1, so the relation is linear.

EXAMPLE 3 Using an algebraic strategy to identify a nonlinear relation

The volume of a cube is the length of one side cubed. Identify the relation between volume and side length as linear or nonlinear.

Joe's Solution

Let s represent the side length of a cube and V represent the volume.
Then, $V = s^3$.

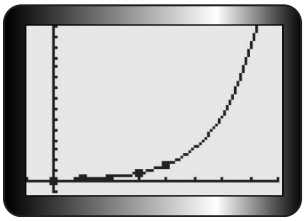
I created an equation for the relation.

The equation is of degree 3, so the relation is nonlinear.

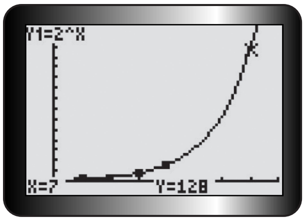
EXAMPLE 4 Evaluating a relation

The number of bacteria, y, in a dish double every hour. An equation for this is $y = 2^x$, where x is time in hours. There is one bacterium at time 0. Predict the number of bacteria at 7 h.

Eva's Solution: Using a strategy involving graphing technology



I graphed the relation. The calculator drew a curve, but really the points shouldn't be connected because the set of data is discrete.



I determined the point at which $x = 7$.

There should be 128 bacteria at 7 h.

Tech Support

For help determining the value of a relation on a graphing calculator, see Appendix B-4.

Wes's Solution: Using a substitution strategy

$$y = 2^x$$
$$y = 2^7$$
$$y = 128$$

I substituted 7 for x in the equation, since this was the number of hours that had gone by.

Time (h)	Number of Bacteria
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128

I used a table of values to see if my answer was reasonable.

I doubled the number of bacteria each hour.

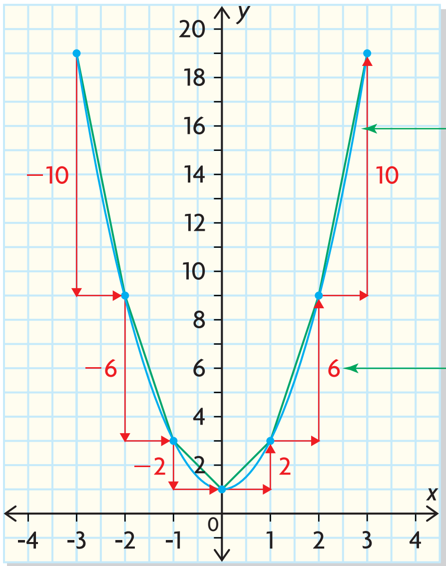
I extended the table to 7 h.

At 7 h, there will be 128 bacteria.

EXAMPLE 5 Thinking about slopes of nonlinear relations

Calculate the slopes of some line segments joining points on the graph of $y = 2x^2 + 1$. How do these compare to the slopes of segments joining points on the graph of a line?

David's Solution



I drew line segments joining points on the graph that had *x*-coordinates 1-unit apart. That meant that for each segment, the run was 1.

Then I calculated the rise for each line segment. Since the run is 1 in each case, the rise is equal to the slope of each segment.

I knew that the slopes of any line segment on a linear graph would be constant.

The slopes are not constant and that is different from the slopes of segments joining points on a linear relation.

For this relation, the farther pairs of points are from 0, the steeper the slope becomes.

In Summary

Key Ideas

- Some relations are nonlinear.
- If a relation is nonlinear, then the following are true:
  - The graph is not a straight line.
  - The first differences are not constant.
  - The degree of its equation is not 1.

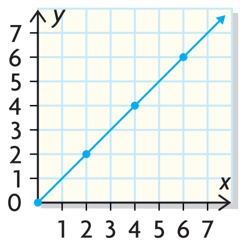
Need to Know

- In a nonlinear relation, the slope between pairs of points is not constant.

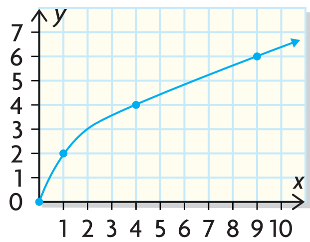
CHECK Your Understanding

1. Identify each relation as linear or nonlinear. Explain how you know.

a)



b)



2. The area of a circle of radius r is $A = \pi r^2$. Identify this relation as linear or nonlinear. Explain.

PRACTISING

3. Identify each relation as linear or nonlinear.

a)

x	y
−3	9
−2	4
−1	1
0	0

b)

x	y
5	1
6	2
7	3
8	4

c)

x	y
1	0.25
2	0.50
3	0.75
4	1.00

4. Josie ran these distances while training for a marathon.

K

Time (min)	Distance Run (km)
0	0
10	2
20	4
30	6

a) Would you choose time or distance as the independent variable? Explain.

b) Identify the relation between distance and time as linear or nonlinear. Explain how you know.

5. This pattern is made from squares with sides of 1 cm.

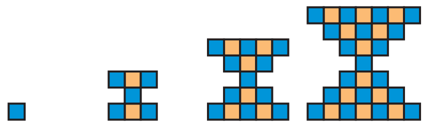


figure 1 figure 2 figure 3 figure 4

a) Use a table to show the perimeter of each figure in terms of its figure number.
b) Determine the perimeter of figure 12. Explain your reasoning.
c) Identify the relation in part a) as linear or nonlinear. Explain how you know.
d) Use a table of values to show the number of blue squares in terms of the figure number.
e) Determine the number of blue squares in figure 12. Explain your reasoning.
f) Identify the relation in part d) as linear or nonlinear. Explain how you know.

6. Identify each relation as linear or nonlinear. Explain your reasoning.

a) the relation between the number of circles in each figure and the figure number

b) the relation between the number of stars in each figure and the figure number

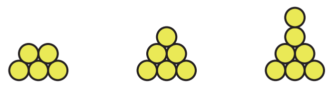


figure 1 figure 2 figure 3

figure 1 figure 2 figure 3 figure 4

7. The relation between kilometres driven, k, and the amount of gasoline, G, (in litres) in the tank of a hybrid car is $G = 80 - 0.2k$.
a) Identify this relation as linear or nonlinear. Explain how you know.
b) Use either a graph or a table to confirm your answer in part a).

8. The amount of cucumbers you can grow in a season depends on the amount of rainfall you get. This relation is represented by the equation $C = 0.006(R + 20)$, where R is the rainfall in millimetres and C is the cucumber yield in kilograms per square metre.
a) Identify this relation as linear or nonlinear. Explain how you know.
b) Use either a graph or a table of values to confirm your answer in part a).

9. When a piece of paper is folded in half, one crease line and two sections of paper are created. The paper is then folded in half again and again, each time increasing the number of crease lines by 1. Identify the relation between the number of creases and the number of sections of paper as linear or nonlinear. Justify your answer.

10. **A** A large hailstone falls from a cloud 5000 m above the ground. This table shows its altitude at different times. About how many seconds will it take for the hailstone to hit the ground? How do you know?

Time (s)	0	5	10	15	20
Altitude (m)	5000	4875	4500	3875	3000

11. **C** Each pattern represents a relation between the figure number and the number of red triangles needed to make it.

a) Which of these patterns are linear and which are nonlinear relations?

b) Explain how you know.

	Pattern 1	**Pattern 2**	**Pattern 3**
Figure 1	♦	♦	♦
Figure 2	♦♦	♦♦ ♦♦	♦ ♦
Figure 3	♦♦♦	♦♦♦ ♦♦♦ ♦♦♦	♦♦ ♦♦

Extending

12. Calculate the first, second, and third differences for each relation. What is the connection between the degree of the equation and the differences?

a) $y = x^2$

b) $y = x^3$

c) $y = -2x^2$

d) $y = 4x^3$

e) $y = \frac{1}{3}x^2 + 2x - 1$

f) $y = -2x^3 + x$

13. Identify each relation as linear or nonlinear. Use a graph or a table to justify your answer.

a) $y = (x + 1)(x - 2)$

b) $y = 2(x - 1)(x + 3)$

c) $y = x^{\frac{1}{2}}$

d) $y = \sqrt{x}$

3 Chapter Review

FREQUENTLY ASKED Questions

Q: How can you graph a linear relation in the form $Ax + By = C$?

Study Aid

- See Lesson 3.4.
- Try Chapter Review Questions 6 and 7.

A1: Plot and join x- and y-intercepts in a straight line, which are found by setting $y = 0$ and $x = 0$ in turn.

EXAMPLE

Graph $3x - 4y = 12$.

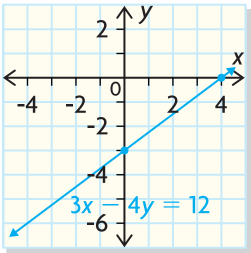
Solution

$$\begin{aligned} (y &= 0) \\ 3x - 4(0) &= 12 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

The x-intercept is 4.

$$\begin{aligned} (x &= 0) \\ 3(0) - 4y &= 12 \\ -4y &= 12 \\ y &= -3 \end{aligned}$$

The y-intercept is -3.



A2: Plot and join ordered pairs (x, y) from a table of values you create.

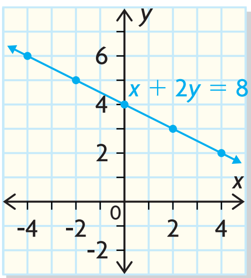
EXAMPLE

Graph $x + 2y = 8$.

Solution

Choose numbers for x and create a table of values.

$x + 2y = 8$					
x	-4	-2	0	2	4
y	6	5	4	3	2



Q: How can you determine whether a relation is nonlinear?

Study Aid

- See Lesson 3.5.
- Try Chapter Review Questions 9, 10, and 11.

A: A relation is nonlinear if its first differences are not constant, its graph is not a straight line, or the degree of its equation is a value other than 1.

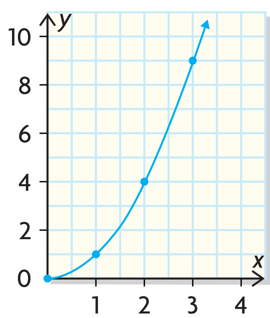
EXAMPLE

Identify the relation $y = x^2$ as linear or nonlinear.

Solution

x	y	Δy
1	1	
		$4 - 1 = 3$
2	4	
		$9 - 4 = 5$
3	9	

nonlinear: first differences not constant



nonlinear: graph curved

$y = x^2$

nonlinear: equation of degree 2

PRACTICE Questions

Lesson 3.1

1. Represent each relation in another form.

a)

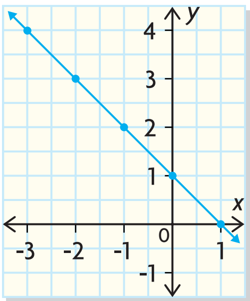
x	y
1	0
3	2
5	4

b)

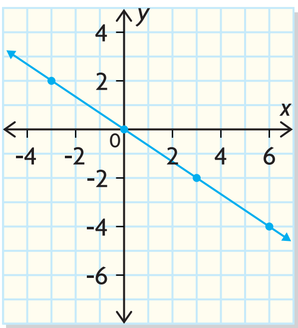
x	y
-1	1
2	-2
5	-5

c) $y = 2x + 4$

d)



e)



f) $3x - 2y = 12$

Lesson 3.2

2. a) Identify each relation in question 1 as a direct or partial variation. Explain.

b) Solve each relation in question 1 for $x = 7$.

Lesson 3.3

3. Calculate the slope of each line.

a) The rise is 4 and the run is 5.

b) $\Delta y = 8$ when $\Delta x = 2$.

c) The change in x is 6 and the change in y is 10.

d) The line passes through points (2, 7) and (6, -1)

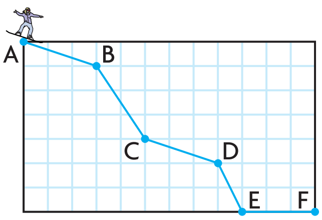
e) The first differences are -5 when the change in x is 1.

4. Kristina is snowboarding down this hill.

a) On which segment will she go fastest? Why?

b) On which segment will she go slowest? Why?

c) Prove your answers to parts a) and b) mathematically.



5. Determine two more ordered pairs that lie on each line.

a) The rise is 3, the run is 4, and (2, -5) is on the line.

b) The slope is $\frac{2}{3}$ and the y-intercept is (0, 5).

c) The slope is $-\frac{3}{5}$ and the x-intercept is (3, 0).

d) $\Delta y = 5$, $\Delta x = 2$, and $(-1, -1)$ is on the line.

Lesson 3.4

6. a) Graph $y = \frac{2}{3}x - 4$

b) Graph $3x - 6y = 12$

7. A rectangle has a perimeter of 210 cm.

a) Explain why $2L + 2W = 210$ models the case. What are L and W?

b) Graph $2L + 2W = 210$.

c) Is the set of data discrete or continuous? Explain.

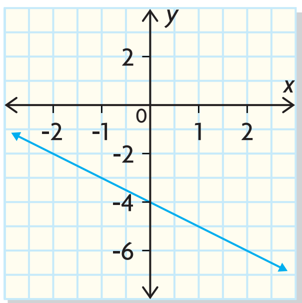
d) Determine two combinations of length and width for this rectangle.

8. Is $x = 5$ the x-intercept of $2x - 3y = 10$? Explain how you know.

Lesson 3.5

9. Identify each relation as linear or nonlinear. Explain your reasoning.

a)

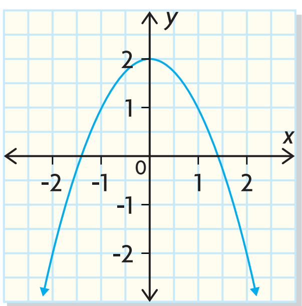


b) $y = 0.25x - 3$

c)

x	−3	−2	−1
y	2	3	4

d)



e) $y^2 = 4 - x^2$

f)

x	−3	−2	−1
y	27	8	1

10. A ball is hit straight up into the air. The table shows its height at various times.

Time (s)	Height (m)
0	1
1	26
2	41
3	46
4	41
5	26
6	1

a) Identify the relation between height and time as linear or nonlinear.

b) Graph the data.

c) Estimate the height of the ball at 1.5 s.

d) Estimate the time at which the height of the ball is 44 m.

e) Determine the time at which the ball hits the ground.

f) Determine the maximum height of the ball.

11. The table shows the value y, in dollars, of a rare coin that is x years old.

x	y
0	0.25
10	750.25
20	1500.25
30	2250.25

a) Is this relationship linear or nonlinear?

b) Graph the data.

c) Find the equation of this relationship.

d) Use the equation to find the value of the coin after 15 years.

3 Chapter Self-Test

1. A fruit stand sells apples for \$0.25 each.
   a) Describe the relation between cost and number of apples bought using a graph, a table, or an equation.
   b) Which variable is independent and which is dependent?
   c) Is the set of data continuous or discrete?
   d) Determine the cost of 150 apples.

2. Gill rents a car for \$45/day plus \$0.15/km. Is the relation between distance and cost linear? Use a table to support your answer.

3. Which choice best describes the relation between distance and time?

Distance (m)	0	4	8	12	16	20	24
Time (s)	0	1	2	3	4	5	6

   A. a linear, direct variation
   B. a linear, partial variation
   C. a nonlinear, direct variation
   D. a nonlinear, partial variation

4. What is the slope of the line between points (0, 4) and (3, −1)?

   A. $-\frac{3}{5}$ **B.** $\frac{3}{5}$ **C.** $-\frac{5}{3}$ **D.** $\frac{5}{3}$

5. What is the rate of change of the stretch of a spring with a weight attached?

Mass (g)	1	2	3	4
Stretch (cm)	5	10	15	20

   A. 5 cm/g **B.** 1 g/5 s **C.** 1 cm/5 g **D.** 5 g/s

6. George is going 6 km on foot. He can run at 4 km/h and walk at 2 km/h.
   a) Explain why $4x + 2y = 6$ models the distance he will travel. What do x and y represent?
   b) How long would it take him to run all the way and to walk all the way?
   c) Graph the combinations of times that he could walk and run.
   d) Determine three combinations of times that George could walk and run.

7. Identify the relation between the figure number and the number of squares as linear or nonlinear. Use a table of values, a graph, and an equation to support your answer.

   a)

figure 1 figure 2 figure 3

   b)

figure 1 figure 2 figure 3

3 Chapter Task

Choosing a Car

Atul is buying a new car. He is deciding between a four-cylinder car and a hybrid crossover car. He wants to choose the car with the better fuel economy.

The four-cylinder car holds 75 L and uses 14 L for every 100 km travelled.

The fuel consumption for the hybrid car is given by $y = 80 - 0.20x$, where y is the amount of fuel in the tank after x kilometres have been driven.

Task Checklist

- ✔ Did you draw and label your graphs?
- ✔ Did you explain your solutions?
- ✔ Did your solutions answer the questions?
- ✔ Did you use appropriate math language?

? Which car should Atul buy?

A. Make a table of values for each car.

B. Use the first differences to compare the rates of change in the fuel volume for each vehicle.

C. Graph the relation between the kilometres driven and the fuel remaining for each car.

D. What are the x- and y-intercepts and what do they mean?

E. What does the slope mean?

F. Use terms from this list to describe the relation between the fuel consumption of each car and the distance driven.

- partial variation
- direct variation
- discrete set of data
- continuous set of data
- linear relation
- nonlinear relation
- independent variable
- dependent variable
- rate of change

G. Write an equation relating the number of kilometres driven and the fuel remaining for the four-cylinder car.

H. Calculate the amount of fuel remaining after driving 250 km for each car.

I. Recommend to Atul which car he should buy. Justify your answer.